# Precalculus Learning Goals - Week 5

This week we'll finish our section on **Functions – Properties and Examples**. Then we'll go on and study **Logarithmic Functions** and **Polynomials** in more detail.

As a reminder, the general goals for the section **Functions – Properties and Examples** are as follows. At the end of this section, students should be able to:

- *Be comfortable with the language, notation, and pictures of functions, as well as be able to translate between them.*
- *Know several examples of functions and their basic properties, both mathematical and "real-world."*
- *Be able to generate new functions from old through the standard function operations.*

**More specifically**, at the end of this week you should be able to:

- Determine if a given function has an inverse based on a formula, graph, table, written description, or other presentation.
- Compute the inverse of a function algebraically.
- Compare the graphs of a function and its inverse.

**Sample Problems.** Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- Compute the inverse of the function $f(x) = \frac{x-2}{3x+4}$.
- Does the function $g(x) = x^3 - x$ have an inverse? Hint: can you factor this function? What does that tell you about its graph?
- Let $h$ be the function given by the following graph (graph omitted). Does $h$ have an inverse? If so, sketch a graph of the inverse of $h$. If not, explain why not.

With all the general function knowledge under our belts, we'll start examining **Logarithmic Functions** in more detail. The general goals for the section **Logarithmic Functions** are as follows. At the end of this section, students should be able to:

- *Define logarithms.*
- *Compute with logarithms, including using rules of logarithms.*
- *Know major properties of graphs of logarithms.*

**More specifically**, at the end of this week you should be able to:

- Define a logarithmic function as the inverse of an exponential function.
- Compute values of logarithmic functions (or estimate for "not nice" numbers).
- State the rules of logarithms.
- Use rules of logarithms to rewrite expressions.
- Convert a logarithmic equation into an exponential equation and vice versa.
- Use the change of base formula to rewrite expressions and compute logarithms.

**Sample Problems.** Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- What is $\log_4(8)$?
- Write $\ln(x) + 2\ln(z) - \ln(4)$ as a single logarithm.
- Expand $\ln(\frac{1}{ex})$ and simplify as much as possible.
- Which is bigger, $\log_2 e$ or $\ln 2$?
- Let's say $\log_3(4) = a$ and $\log_5(3) = b$ and $\log_4(3) = c$ and $\log_4(5) = d$. What is $\log_5(4)$?
- How are the functions $\log_2(x)$ and $2^x$ related?
- Simplify $e^{5\ln(3)}$ as much as possible.
- Say you know that $\ln(a) = 5$. What can you say about $\ln(ae^3)$?

Finally this week, we'll study **Polynomial Functions.** The general goals for this section are as follows. At the end of this section, students should be able to:

- *Know polynomial terminology and basic properties.*
- *Divide polynomials.*
- *Understand and apply the Intermediate Value Theorem.*

**More specifically**, at the end of this week you should be able to:

- Define *polynomial, root, degree, coefficient, leading term, end behavior.*
- Find the quotient of two polynomials.
- Define *continuous* function intuitively.
- State the Intermediate Value Theorem.
- Apply the Intermediate Value Theorem to find zeros.
- Derive implications of the Intermediate Value Theorem.

**Sample Problems.** Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- What is the degree of the polynomial $p(x) = (x+3)^2(4x-2)(2-x)^5$? As $x$ approaches negative infinity, does $p(x)$ approach $-\infty, 0$, or $\infty$?
- What is $x^3 - 2x + 5 \div 2x - 1$?
- Find a function that is continuous that is not a polynomial.
- True or False: you were once exactly 3 feet tall. Explain your answer.
- True or False: there was once a time when your weight in pounds equaled your height in inches. Explain your answer.
- True or False: during his record-setting game when he scored 100 points, there must have been a time when Wilt Chamberlain had scored exactly 35 points. Explain your answer.
- True or False: the function $x^{99} - x^2 - 5$ has a root in $[0, 2]$. Explain your answer.